Composition of Switching Lattices and Autosymmetric Boolean Function Synthesis

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NANOxCOMP

Introduction

CMOS technology

- Transistor size have been shrunk for decades
- The trend reached a critical point

The Moore's Law era is coming to end

New emerging technologies

- Biotechnologies, molecular-scale self-assembled systems
- Graphene structures
- Switching lattices arrays

These technologies are in an early state

A novel synthesis approach is necessary, focused on the properties of the devices
Synthesis efficiency can be the main factor for a technology choice

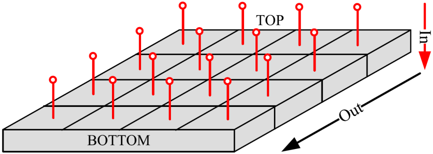
We focus our work on Synthesis for Switching Lattices

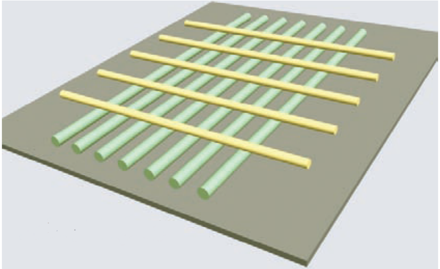
Switching Lattices

How Switching Lattices are made

Nanowires are one of the most promising technologies

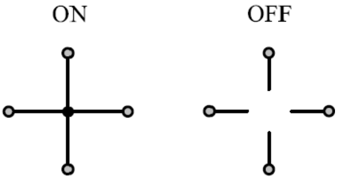
- Nanowire circuits can be made with **self-assembled structures**
- **pn-junctions** are built crossing *n*-type and *p*-type nanowires
- **Low** V_{in} voltage makes *p*-nanowires conductive and *n*-nanowires resistive
- **High** V_{in} voltage makes *n*-nanowires conductive and *p*-nanowires resistive

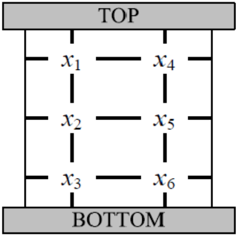




Switching Lattices are **two-dimensional** array of **four-terminal** switches

- When switches are **ON** all terminals are connected, when **OFF** all terminals are disconnected
- Each switch is controlled by a boolean literal, **1** or **0**
- The boolean function f is the SOP of the literals along each path from **top** to **bottom**
- $f = x_1x_2x_3 + x_1x_2x_5x_6 +$ $+x_4x_5x_2x_3 + x_4x_5x_6$

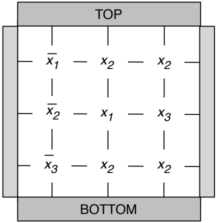




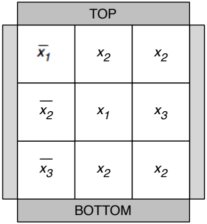
From Crossbars to Lattices

For an easier representation the **crossbars** are converted to **lattices**:

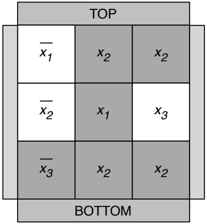
- A ‘checkerboard’ notation is used
- Darker and white sites represent **ON** and **OFF**
- a), b): the 4-terminal switching network and the lattice describing $f = \overline{x}_1\overline{x}_2\overline{x}_3 + x_1x_2 + x_2x_3$
- c), d): the lattice evaluated on inputs (1,1,0) and (0,0,1)



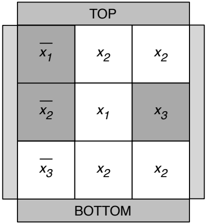
(a)



(b)



(c)



(d)

The synthesis methods

Altun-Riedel, 2012

- Synthesizes f and f^D from **top to bottom** and **left to right**
- It produces lattices with size growing **linearly** with the SOP
- Time **complexity is polynomial** in the number of products

TOP

LEFT					RIGHT
	$\bar{x}_6$	$\bar{x}_8$	$\bar{x}_6$	x_4	
	$\bar{x}_8$	$\bar{x}_8$	$\bar{x}_5$	x_4	
	$\bar{x}_6$	$\bar{x}_7$	$\bar{x}_6$	x_4	
	$\bar{x}_7$	$\bar{x}_7$	$\bar{x}_5$	x_4	
	$\bar{x}_6$	$\bar{x}_5$	$\bar{x}_6$	x_4	
	x_1	x_1	x_1	x_1	
	$\bar{x}_2$	$\bar{x}_2$	$\bar{x}_2$	$\bar{x}_2$	
	x_3	x_3	x_3	x_3	

BOTTOM

Gange-Søndergaard-Stuckey, 2014

- f is synthesized from **top to bottom**
- The synthesis problem is formulated as a **satisfiability problem**, then the problem is solved with a SAT solver
- The synthesis method searches for better implementations starting from an upper bound size
- The synthesis loses the possibility to generate both f and f^D

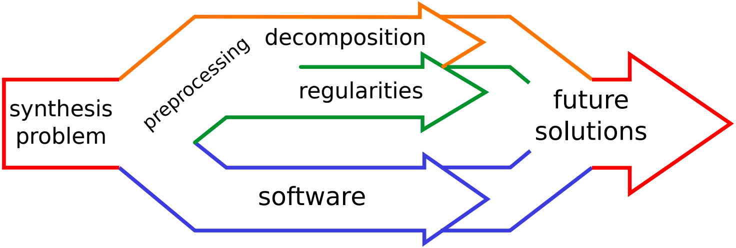
TOP

$\bar{x}_4$	x_6	x_7
x_2	x_5	x_8
$\bar{x}_1$	x_2	x_6
$\bar{x}_3$	0	$\bar{x}_6$

BOTTOM

$$f = \overline{x}_8\overline{x}_7\overline{x}_6x_3\overline{x}_2x_1 + \overline{x}_8\overline{x}_7\overline{x}_5x_3\overline{x}_2x_1 + x_4x_3\overline{x}_2x_1$$

Approach to the synthesis problem



To optimize lattice synthesis there are different approaches, but common goals:

- Produce optimal-size lattices
- Reduce synthesis time
- Create efficient methods for sub-optimal lattice synthesis

Use of sub-optimal lattices when optimal synthesis requires too much computing time or memory

Decomposition Techniques and Lattices

The logic synthesis of 4-terminal switches can be very **computational intensive**

Boolean function **decomposition techniques**

- **decompose** a function according to a given decomposition scheme
- implement the **decomposed blocks** into a single or multiple lattices
- decomposed functions have **less variables** and/or a **smaller on-set**
- the implementation may be **smaller** and the synthesis **less computational intensive**

We use a preprocessing technique that exploits the properties of a the **Autosymmetric** boolean functions

Autosymmetric functions

Regularities: autosymmetric boolean functions

- Consider a Boolean function $f : \{0,1\}^n \to \{0,1\}$: the function f is *closed under* a vector $\boldsymbol{\alpha} \in \{0,1\}^n$, if for each vector $\boldsymbol{w} \in \{0,1\}^n$, $\boldsymbol{w} \oplus \boldsymbol{\alpha} \in f$ if and only if $\boldsymbol{w} \in f$.
- The set $L_f = \{\boldsymbol{\beta}\colon f \text{ is closed under } \boldsymbol{\beta}\}$ is a **vector subspace** of $(\{0,1\}^n, \oplus)$. The set L_f is called the **vector space** of f.

- **Definition:** A completely specified Boolean function f is *k-autosymmetric* if its vector space L_f has dimension k.
- **Definition:** Let V be a vector subspace of $(\{0,1\}^n, \oplus)$. The set $A = \boldsymbol{\alpha} \oplus V$, $\boldsymbol{\alpha} \in \{0,1\}^n$, is an *affine space* over V with *translation point* $\boldsymbol{\alpha}$.

The points of f can be partitioned into $\ell = |f|/2^k$ disjoint sets, where $|f|$ denotes the number of points of f; all these sets are affine spaces over L_f.

$$f = \bigcup_{i=1}^{\ell} (\boldsymbol{w}^i \oplus L_f)$$

Autosymmetric functions

Autosymmetric functions can be **reduced to "equivalent, but smaller" functions** if f is *k-autosymmetric*,

- f_k is a function over $n - k$ variables, y_1, y_2, ..., y_{n-k}, such that

$$f(x_1, ..., x_n) = f_k(y_1, ..., y_{n-k})$$

- y_i is an EXOR combination of a subset of x_i's.
- These combinations are EXOR(Xi), where $X_i \subseteq X$
- $y_i = EXOR(X_i)$, $i = 1, ..., n - k$, are called **reduction equations**
- f_k is called a **restriction of f**

f_k is "equivalent" to, but **smaller than f**, and has $|f|/2^k$ points only.

Example of autosymmetric function decomposition

- $f = \{0000, 0001, 0010, 0011, \ 0100, 0101, 0110, 0111, 1000, 1011, 1101, 1110\}$
- Vector space $L_f = \{0000, 0011, 0101, 0110\}$
- Canonical variables x_2 and x_3 (independent variables on L_f).

- We can build f_2 by taking $f_{x_2=0,x_3=0} = \{00, 01, 10\}$: $f_2(y_1, y_2) = \overline{y_1 y_2}$.
- The homogeneous system whose solutions are $\{0000, 0011, 0101, 0110\}$ is:

$$\begin{cases} x_1 & = 0 \\ x_2 \oplus x_3 \oplus x_4 & = 0 \end{cases}$$

Autosymmetric boolean functions have already **studied and algebraically characterized**
The space L_f, the function f_k and the reduction equation can be **calculated in a polynomial time**

Example of autosymmetric function decomposition

- $f = \{0000, 0001, 0010, 0011, 0100, 0101, 0110, 0111, 1000, 1011, 1101, 1110\}$
- Vector space $L_f = \{0000, 0011, 0101, 0110\}$
- Canonical variables x_2 and x_3 (independent variables on L_f).

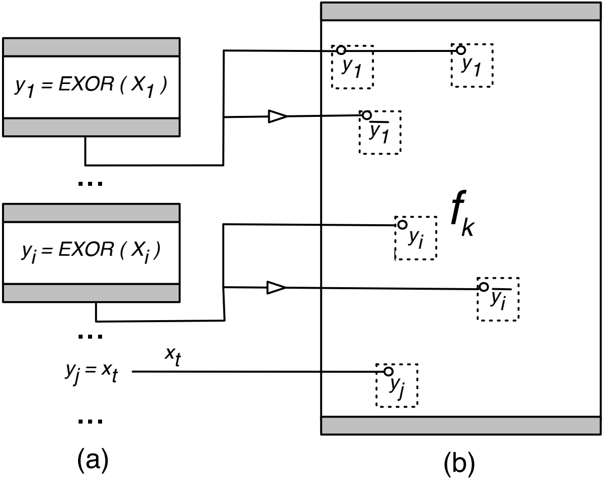
Thus the reduction equations are given by

$$y_1 = x_1 \tag{1}$$
$$y_2 = x_2 \oplus x_3 \oplus x_4 . \tag{2}$$

f can be represented as:

$$f(x_1, x_2, x_3, x_4) = f_2(y_1, y_2) = \overline{y_1 y_2} = \overline{x}_1(\overline{x_2 \oplus x_3 \oplus x_4}) .$$

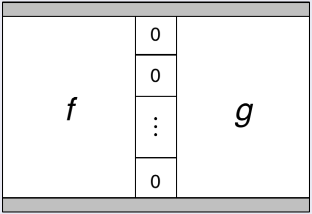
Lattice implementation of autosymmetric functions



Disjunction and conjunction of lattices

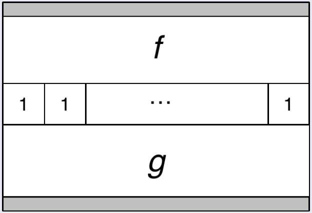
$f + g$

- separate the paths from top to bottom for f and g
- add a column of 0s
- add padding rows of 1s if lattices have different number of rows



$f \cdot g$

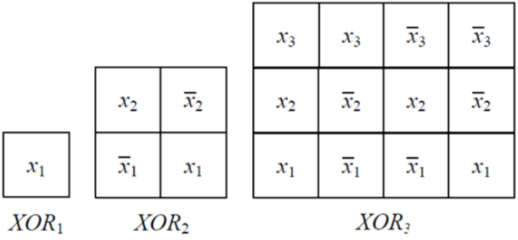
- any top-bottom path of f is joined to any top-bottom path of g
- add a row of 1s
- add padding columns of 0s if lattices have different number of columns



Lattices of EXOR functions

EXOR factors lattices are simple to synthesize

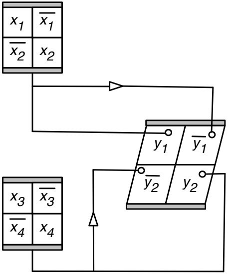
- the dimension of a two-variables EXOR lattice is 2×2
- the dimension of a three-variables EXOR lattice is 4×3



Autosymmetric function: example

- $f(x_1, x_2, x_3, x_4) = x_1 \oplus x_2 \oplus x_3 \oplus x_4$.
- decomposing: $f = g(y_1, y_2) = y_1 \oplus y_2$, where $y_1 = x_1 \oplus x_2$ and $y_2 = x_3 \oplus x_4$
- Multi-lattice: the sum of the areas of the lattices is smaller than the area of the optimum single-lattice

x_1	$\overline{x_1}$	0	x_1	$\overline{x_1}$
$\overline{x_2}$	x_2	0	x_2	$\overline{x_2}$
1	1	0	1	1
x_3	$\overline{x_3}$	0	x_3	$\overline{x_3}$
x_4	$\overline{x_4}$	0	$\overline{x_4}$	x_4



Experiments

- Benchmarks are taken from LGSynth93
- Each benchmark output is considered as a separate boolean function
- A total of 607 functions including 53 autosymmetric functions
- We use a collection of Python scripts and a SAT solver to perform the Gange-Søndergaard-Stuckey synthesis

- The algorithm has been implemented in C
- The experiments have been run on a machine with 16 CPU @2.5 GHz, running Centos 6.6

Auto-symmetric functions decomposition results

F-Name	Altun-Riedel		Gange-Søndergaard-Stuckey	
	standard Row×Col	Decomposed Row×Col + XOR area	standard Row×Col	Decomposed Row×Col + XOR area
add6(0)	**2**×**2**	1×1 + 4	**2**×**2**	1×1 + 4
add6(1)	6×6	**3**×**3 + 4**	5×3	**3**×**3 + 4**
dekoder(0)	4×2	**3**×**1 + 4**	4×2	**3**×**1 + 4**
dekoder(1)	3×2	2×1 + 4	3×2	2×1 + 4
rd53(1)	10×10	6×5 + 16	–	**4**×**3 + 16**
sqn(0)	17×16	7×7 + 8	–	**3**×**5 + 8**

– : Gange-Søndergaard-Stuckey synthesis does not finish in 10min

Conclusions

- **Smaller lattices**: at least 53% of area reduction in 48% of functions.
- **Affordable computing time**, in some cases is possible to find a solution in less time than the optimum one.
- Some decomposed functions has **smaller total area** w.r.t. the lattice size in optimum case.
- Increase the number of lattices and the final lattice has more complex signal routing.

Thank you!