BLG231 – HW1 Solutions

1) a) $(100111,10111)_2 = 2^5+2^2+2^1+2^0+2^{-1}+2^{-3}+2^{-4}+2^{-5} = (39,71875)_{10}$

$(\underbrace{100}_{4}\underbrace{111}_{7},\underbrace{101}_{5}\underbrace{110}_{6})_2 = (47,56)_8$

$(\underbrace{10}_{2}\underbrace{0111}_{7},\underbrace{1011}_{B}\underbrace{1000}_{8})_2 = (27,B8)_{16}$

b) $(\underset{111}{7}\underset{010}{2},\underset{011}{6})_8 = (\underbrace{11}_{3}\underbrace{1010}_{A},\underbrace{1100}_{C})_2 = (3A,C)_{16} = 7.8^1+2.8^0+6.8^{-1} = (58,75)_{10}$

c) $(C3,AD5)_{16}$ (1100 0011 1010 1101 0101) $= (\underbrace{11}_{3}\underbrace{000}_{0}\underbrace{011}_{3},\underbrace{101}_{5}\underbrace{011}_{3}\underbrace{010}_{2}\underbrace{101}_{5})_2 = (303,5325)_8 = 12.16^1+3.16^0+10.16^{-1}+13.16^{-2}+5.16^{-3}$

$= (195,677001953125)_{10}$

d) $(\underbrace{1101}_{D}\,\underbrace{1100}_{C}\,\underbrace{1010}_{A}\,\underbrace{1010}_{A}\,\underbrace{1010}_{A}\,\underbrace{1000}_{8}\,\underbrace{1011}_{B}\,\underbrace{1111}_{F}\,\underbrace{1010}_{A}\,\underbrace{0011}_{3}\,\underbrace{1111}_{F})_2 = (DCAAA8BFA3F)_2$

2) a) $\overline{x_1x_2+x_2x_3+x_3x_4} = (\bar{x}_1+\bar{x}_2)(\bar{x}_2+\bar{x}_3)(\bar{x}_3+\bar{x}_4)$

$= \bar{x}_1\bar{x}_2\bar{x}_3+\bar{x}_1\bar{x}_2\bar{x}_4+\bar{x}_1\underline{\bar{x}_3\bar{x}_3}+\bar{x}_1\bar{x}_3\bar{x}_4+\underline{\bar{x}_2\bar{x}_2}\bar{x}_3+\underline{\bar{x}_2\bar{x}_2}\bar{x}_4+\bar{x}_2\underline{\bar{x}_3\bar{x}_3}+\bar{x}_2\bar{x}_3\bar{x}_4$

$= \cancel{\bar{x}_1\bar{x}_2\bar{x}_3}+\cancel{\bar{x}_1\bar{x}_2\bar{x}_4}+\bar{x}_1\bar{x}_3+\cancel{\bar{x}_1\bar{x}_3\bar{x}_4}+\bar{x}_2\bar{x}_3+\bar{x}_2\bar{x}_4+\cancel{\bar{x}_2\bar{x}_3}+\cancel{\bar{x}_2\bar{x}_3\bar{x}_4}$

$= \bar{x}_1\bar{x}_3+\bar{x}_2\bar{x}_3+\bar{x}_2\bar{x}_4$ // $\Rightarrow$ 6 literals

b) $\overline{\bar{x}_1x_2x_3+x_1x_4} = (x_1+\bar{x}_2+\bar{x}_3)(\bar{x}_1+\bar{x}_4) = \cancel{x_1\bar{x}_1}+x_1\bar{x}_4+\bar{x}_2\bar{x}_1+\cancel{\bar{x}_2\bar{x}_4}+\bar{x}_1\bar{x}_3+\cancel{\bar{x}_3\bar{x}_4}$

$= x_1\bar{x}_4+\bar{x}_1\bar{x}_2+\bar{x}_1\bar{x}_3$ // $\Rightarrow$ 6 literals

c) $\overline{x_1\bar{x}_2x_3+x_1\bar{x}_4+x_2x_3\bar{x}_4} = (\bar{x}_1+x_2+\bar{x}_3)(\bar{x}_1+x_4).(\bar{x}_2+\bar{x}_3+x_4)$

$= \cancel{\bar{x}_1}\bar{x}_1\bar{x}_2+\cancel{\bar{x}_1}\bar{x}_1\bar{x}_3+\cancel{\bar{x}_1}\bar{x}_1x_4+\bar{x}_1x_4\bar{x}_2+\bar{x}_1x_4\bar{x}_3+\bar{x}_1x_4\cancel{x_4}+\cancel{x_2\bar{x}_1\bar{x}_2}+x_2\bar{x}_1\bar{x}_3+x_2\bar{x}_1x_4+\cancel{x_2x_4\bar{x}_2}+x_2x_4\bar{x}_3+x_2x_4\cancel{x_4}$

$+\bar{x}_3\bar{x}_1\bar{x}_2+\bar{x}_3\bar{x}_1\cancel{\bar{x}_3}+\bar{x}_3\bar{x}_1x_4+\bar{x}_3x_4\bar{x}_2+\bar{x}_3x_4\cancel{\bar{x}_3}+\bar{x}_3x_4\cancel{x_4}$

$= \bar{x}_1\bar{x}_2+\bar{x}_1\bar{x}_3+\bar{x}_1x_4+\cancel{\bar{x}_1\bar{x}_2x_4}+\cancel{\bar{x}_1\bar{x}_3x_4}+\cancel{\bar{x}_1x_4}+\cancel{\bar{x}_1x_2\bar{x}_3}+\cancel{\bar{x}_1x_2x_4}+\cancel{x_2\bar{x}_3x_4}+x_2x_4+\cancel{\bar{x}_1\bar{x}_2x_3}+\cancel{\bar{x}_1\bar{x}_3}+$

$\cancel{\bar{x}_1\bar{x}_3x_4}+\cancel{\bar{x}_2\bar{x}_3x_4}+\bar{x}_3x_4$

$= \bar{x}_1\bar{x}_2+\bar{x}_1\bar{x}_3+\cancel{\bar{x}_1x_4}+x_2x_4+\bar{x}_3x_4 = \bar{x}_1\bar{x}_2+\bar{x}_1\bar{x}_3+x_2x_4+\bar{x}_3x_4 \Rightarrow$ 8 literals

3-)

X_1	X_2	X_3	X_4	Out
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	0
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	1
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

assigned arbitrarily

Each arrow corresponds to a transition in just one of the inputs.

2- d) $\overline{x_1x_2\bar{x}_3 + x_1\bar{x}_2x_3 + \bar{x}_1x_2x_3 + \overline{x_1}\,\overline{x_2}\,\overline{x_3}} = \overline{x_1(\underbrace{x_2\bar{x}_3+\bar{x}_2x_3}_{a}) + \bar{x}_1(\underbrace{x_2x_3+\overline{x_2}\,\overline{x_3}}_{\bar{a}})}$

$= \overline{x_1\cdot a + \bar{x}_1\cdot\bar{a}} = (\bar{x}_1+\bar{a})(x_1+a) = \cancel{\bar{x}_1x_1} + \bar{x}_1a + ax_1 + \cancel{\bar{a}a}$

$= \bar{x}_1(x_2\bar{x}_3+\bar{x}_2x_3) + x_1(x_2x_3+\bar{x}_2\bar{x}_3)$

$= \bar{x}_1x_2\bar{x}_3 + \bar{x}_1\bar{x}_2x_3 + x_1x_2x_3 + x_1\bar{x}_2\bar{x}_3 \Rightarrow 12$ literals

4) a) SOP form;

$$C_{out} = \bar{A}BC_{in} + A\bar{B}C_{in} + AB\overline{C_{in}} + ABC_{in}$$

$$S = \bar{A}\bar{B}C_{in} + \bar{A}B\overline{C_{in}} + A\bar{B}C_{in} + ABC_{in}$$

POS form;

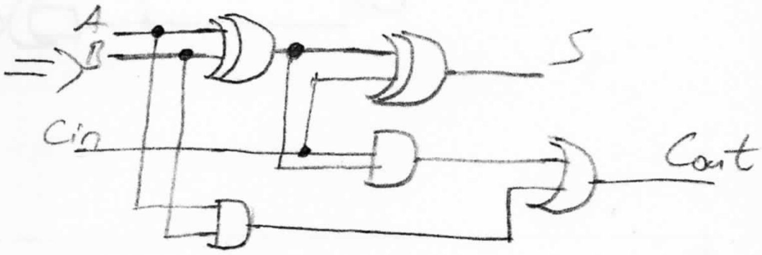
$$C_{out} = (A+B+C_{in}).(A+\bar{B}+\overline{C_{in}}).(\bar{A}+B+\overline{C_{in}}).(\bar{A}+\bar{B}+C_{in}$$

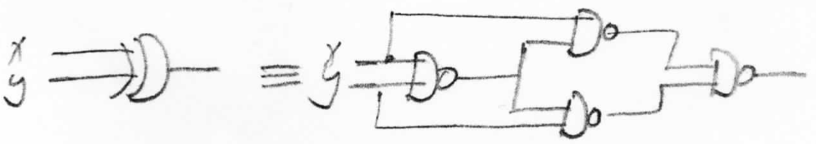
$$S = (A+B+C_{in}).(A+\bar{B}+C_{in}).(A+B+\overline{C_{in}}).(\bar{A}+B+C_{in}$$

b) Using SOP form;

$$S = (A \oplus B) \oplus C_{in}$$

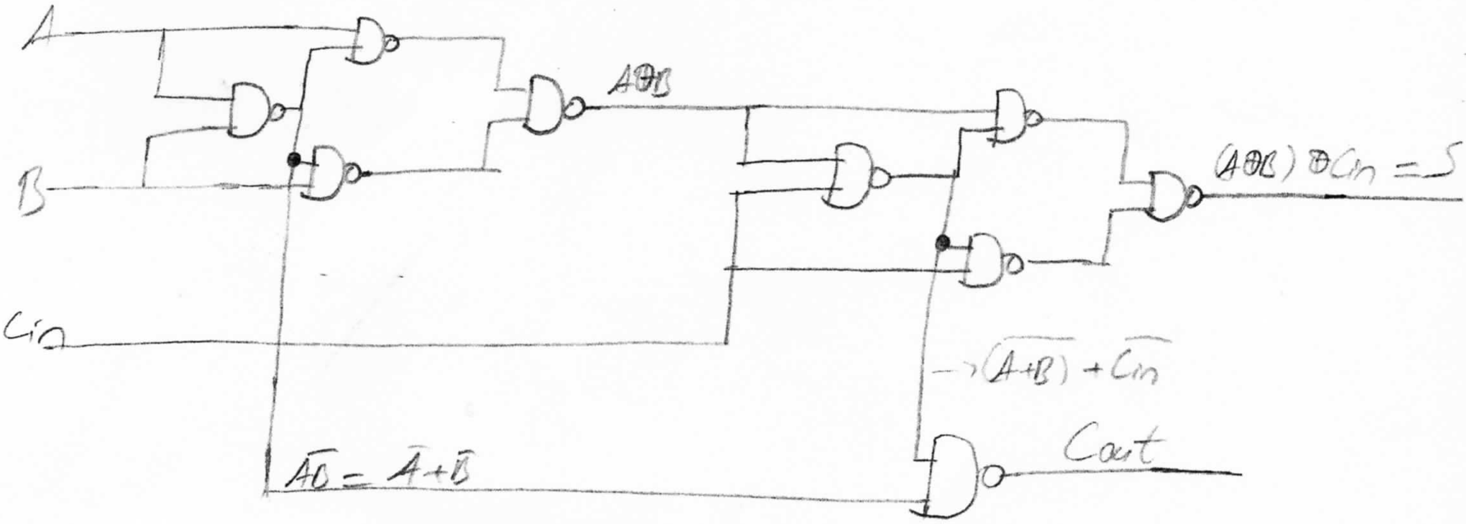
$$C_{out} = AB + (A \oplus B).C_{in}$$





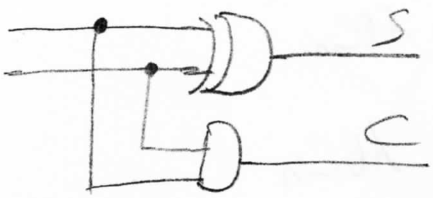
→ use instead of XOR

$$C_{out} = \overline{(\bar{A}+\bar{B}).(\overline{A \oplus B}+\overline{C_{in}})}$$



4 [redacted]

4) c) Construct using two half adder;



$S = A \oplus B$

$C = AB$ $\Rightarrow$