

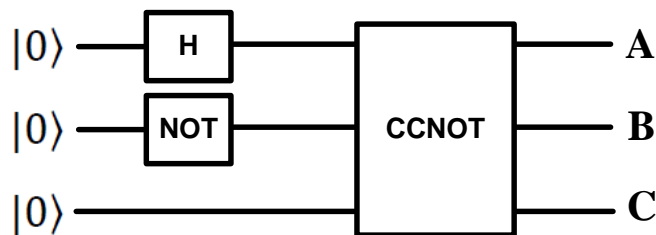
# ELE523E Computational Nanoelectronics Homework 1

Deadline: 17/10/2016 (before 13:30)

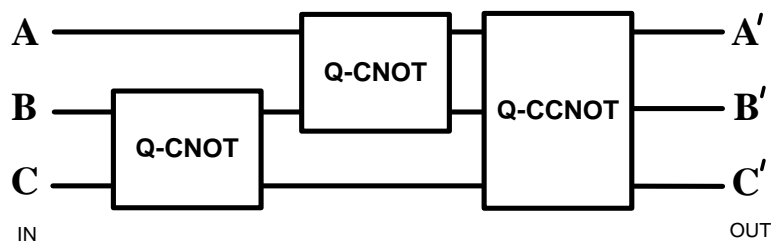
## 1. QUANTUM COMPUTING

a) For the quantum circuit shown below, find the output quantum state and determine the probabilities of each output combination.

- Hint: the output state can be formalized as  $\sum_i \alpha_i |(ABC)_i\rangle$ ; you need to find  $\alpha$  values.



b) Find the truth table of the quantum circuit shown below.



c) Prove that Toffoli (CCNOT) gate is a universal quantum gate (in order to implement any Boolean function).

- Hint: try to implement main Boolean operators
- You are allowed to use  $|0\rangle$  or  $|1\rangle$  as gate inputs.

d) Implement the Boolean function  $f = x_1x_2 + x_1x_3 + x_2x_3x_4$  using *minimum* number of Toffoli gates.

- You are allowed to use  $|0\rangle$  or  $|1\rangle$  as gate inputs.

## 2. REVERSIBLE CIRCUIT DESIGN

a) Implement the following truth tables with reversible circuits using NOT, CNOT, and Toffoli (CCNOT) gates.

IN <u>cba</u>	OUT <u>cba</u>
000	001
001	111
010	000
011	110
100	101
101	100
110	011
111	010

IN <u>cba</u>	OUT <u>cba</u>
000	111
001	001
010	011
011	010
100	110
101	000
110	100
111	101

- b) Implement the following truth table with a reversible circuit using *minimum* number NOT and CNOT gates.

IN	OUT
<u>ba</u>	<u>ba</u>
00	11
01	01
10	10
11	00

- c) Determine the number of input/output bits of a reversible binary multiplier transformed from a 2-bit by 2-bit irreversible multiplier.
- d) Determine the number of input/output bits of a reversible binary multiplier transformed from a 3-bit by 3-bit irreversible multiplier.

### 3. FACTORIZING SEMI-PRIME NUMBERS

- a) Write an algorithm that factorizes *semi-prime numbers*.
- Attach your pseudo and real codes (Matlab, C, etc.) to your homework.
- b) Determine *the worst-case time complexity* of your algorithm. Is it polynomial?
- c) Determine the success rate of your algorithm. Does it always give you the right answer?
- d) To evaluate your algorithm's performance, use semi-prime numbers **15**, **77**, **529**, and **4633** as inputs. Determine the running time of your algorithm for each case.

Grading: 1a)5%, 1b)5%, 1c)10%, 1d)10%  
2a)10%, 2b)10%, 2c)7.5%, 2d)7.5%,  
3a)15%, 3b)10%, 3c)5%, 3d)5%

Note: Return a hard-copy of your homework before the lecture; you can also put your homework under my door.