## Student ID:

Date: 11/1/2017

# BLG231E Digital Circuits <br> FINAL 

Duration: 120 Minutes
Grading: 1) $20 \%$, 2) $15 \%$, 3) $25 \%$, 4) $40 \%$
Exam is in closed-notes and closed-books format; calculators are allowed
For your answers please use the space provided in the exam sheet
GOOD LUCK!

1) Consider a 6-variable Boolean function $f=\boldsymbol{f}_{\mathbf{1}}\left(\boldsymbol{x}_{\mathbf{1}}, \boldsymbol{x}_{\mathbf{2}}, \boldsymbol{x}_{\mathbf{3}}, \boldsymbol{x}_{\mathbf{4}}\right) . \boldsymbol{f}_{\mathbf{2}}\left(\boldsymbol{x}_{4}, \boldsymbol{x}_{5}, \boldsymbol{x}_{\mathbf{6}}\right)$ where $\boldsymbol{f}_{\mathbf{1}}=\Pi$ $(1,2,3,5,7,12,14)$ and $\boldsymbol{f}_{2}=\Pi(3,4,5,6,7)$. Obtain a minimal product-of-sums (POS) expression for $f$.
2) Implement a 4 -variable Boolean function $f\left(x_{1}, x_{2}, x_{3}, x_{4}\right)=\sum(2,4,5,7,8,9,11,13,15)$ using a single 4-to-1 multiplexer and minimal number of two-input NAND gates. Use $\boldsymbol{x}_{\mathbf{1}}$ and $\boldsymbol{x}_{\mathbf{2}}$ as select input lines in the multiplexer. Use only variables $x_{1}, x_{2}, x_{3}, x_{4}$ as inputs (not their negated forms).
3) Consider a flip-flop consisting of four NAND gates, shown below. Suppose that each of the NAND gates has a delay of 2ns. Suppose that initial values of Q and Q' are 0 and 1, respectively.
a) Obtain a minimal sum-of-products (SOP) expression for $\mathbf{Q}$ in terms of previous $Q$, and inputs A and CLK.
b) Sketch the waveforms at the outputs $\mathbf{Q}$ and $\mathbf{Q}^{\prime}$ if the input signals A and CLK shown below are applied.

4) Consider a Mealy machine based sequential circuit having one input $\mathbf{X}$ and one output $\mathbf{Z}$. The output Z will be 0 except the input sequence $\mathbf{1 1 0 1}$ are the last 4 inputs received on X . The circuit has 7 states defined as follows:

| Input <br> Sequence | State |
| :--- | :--- |
| $\ldots \ldots \ldots . .00$ | A |
| $\ldots \ldots . .11$ | B |
| $\ldots \ldots .001$ | C |
| $\ldots \ldots .010$ | D |
| $\ldots \ldots .110$ | E |
| $\ldots \ldots .0101$ | F |
| $\ldots \ldots .1101$ | G |

a) Obtain the state table and perform state reduction. After state reduction how many states remain?
b) Draw the state diagram having the reduced states.
c) Start designing the circuit with assigning 0 's and 1 's to the reduced states. Your design will be based on D-flip-flops. Derive minimal sum-of-products (SOP) Boolean expressions for the next states and the output in terms of the input and the current states.
d) Draw the circuit using minimal number of D-flipflops, and AND, OR, and NOT gates.

EXTRA PAGE FOR SOLUTIONS

