## Student ID:

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# BLG231E Digital Circuits <br> MIDTERM I 

Duration: 120 Minutes
Grading: 1) $15 \%$, 2) $20 \%$, 3) $30 \%$, 4) $20 \%$, 5) $15 \%$
Exam is in closed-notes and closed-books format; calculators are allowed
For your answers please use the space provided in the exam sheet
GOOD LUCK!

1) Consider a 4 -variable Boolean function $\boldsymbol{f}\left(\boldsymbol{x}_{\mathbf{1}}, \boldsymbol{x}_{\mathbf{2}}, \boldsymbol{x}_{\mathbf{3}}, \boldsymbol{x}_{\mathbf{4}}\right)=\prod(0,2,7,8,10,14) ; \boldsymbol{x}_{\mathbf{1}}$ is the most significant bit. Obtain a minimal sum-of-products (SOP) expression for $\boldsymbol{f}$ using a Karnaugh map. Show all prime and essential prime implicants.
2) Consider a 4-variable Boolean function $\boldsymbol{f}\left(\boldsymbol{x}_{\mathbf{1}}, \boldsymbol{x}_{\mathbf{2}}, \boldsymbol{x}_{\mathbf{3}}, \boldsymbol{x}_{\mathbf{4}}\right)=\sum(0,4,5,6,7,8,9,10,11,13,14,15)$; $\boldsymbol{x}_{\boldsymbol{1}}$ is the most significant bit.
a) Using a Quine-McCluskey method, sketch the prime implicant table and show the essential prime implicants in the table.
b) Using the table in a), obtain a minimal sum-of-products (SOP) expression for $f$.
3) Consider a 6-variable Boolean function $f=f_{1}\left(x_{1}, x_{2}, x_{3}, x_{4}\right)+f_{2}\left(x_{4}, x_{5}, x_{6}\right)$ where $f_{1}=\sum$ $(1,2,3,5,7,12,14)-\boldsymbol{x}_{\mathbf{1}}$ is the most significant bit, and $\boldsymbol{f}_{2}=\sum(3,4,5,6,7) \boldsymbol{x}_{\mathbf{4}}$ is the most significant bit.
a) Obtain a minimal sum-of-products (SOP) expression for $f$.
b) Implement $\boldsymbol{f}$ using only two-input NAND (NAND-2) gates; use minimal number of gates. Use only variables as inputs (not their negated forms).
4) Consider a circuit with 6 inputs $\mathbf{a}, \mathbf{b}, \mathbf{A}, \mathbf{B}, \mathbf{C}$, and $\mathbf{D}$, and an output $\mathbf{Q}$.

a) Convert this circuit to a NAND-2 based circuit.
b) If the input $\mathbf{a}$ is always logic 0 , $\mathbf{a}=0$ then simplify your NAND-2 based circuit by deleting unnecessary gates.
5) Consider a 7 -segment display as shown below. It has 7 segments corresponding to 7 outputs $\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}, \mathbf{e}, \mathbf{f}$, and $\mathbf{g}$. If an output is logic 1 then the corresponding segment is illuminated or lit. For example, if $\mathbf{a}=\mathbf{b}=\mathbf{c}=\mathbf{d}=\mathbf{e}=\mathbf{f}=\mathbf{g}=1$ then all segments are lit that shows a digit 8 . Digits $0,1,2,3,4,5,6,7,8$, and 9 are aimed to be displayed such that when there is a binary input corresponding to a digit, the digit is shown in the display.

a) How many inputs should a display at least have? Is there any "don't" care condition?
b) Obtain the truth table.
