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Student ID:

Date: 5/12/2016

ELE523E Computational Nanoelectronics
MIDTERM

Duration: 120 Minutes

Grading:

1) 15% (3% for each right; deduct 1,5% for each wrong), 2) 15%, 3) 10%, 4) 20%, 5) 20%, 6) 20%

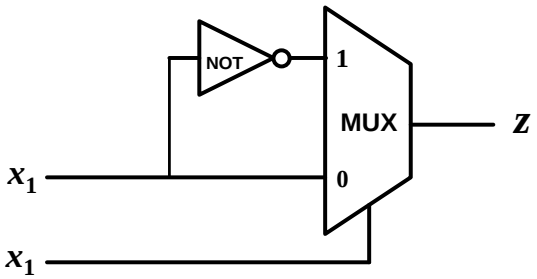
Exam is in closed-notes and closed-books format

For your answers please use the space provided in the exam sheet

GOOD LUCK!

1) Please circle TRUE if you think that the statement is true; FALSE otherwise.

a)



Statement: The **stochastic** expression of $z = (1 - x_1)$.

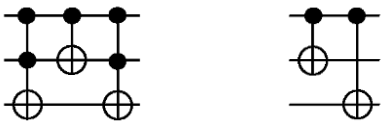
TRUE / FALSE

b) Consider two sets A and B consisting of n consecutive integers such that $A=\{a_1, a_2=a_1+1,\ldots, a_{n-1}=a_1+n-2, a_n=a_1+n-1\}$ and $B=\{b_1, b_2=b_1+1,\ldots, b_{n-1}=b_1+n-2, b_n=b_1+n-1\}$. For example, $A=\{-1, 0, 1, 2\}$ and $B=\{1, 2, 3, 4\}$ where $n=4$.

Statement: The time complexity of testing whether A and B intersects (at least one common integer in A and B) is $O(1)$.

TRUE / FALSE

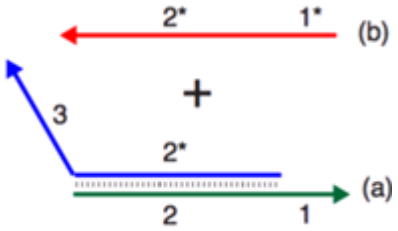
c) Quantum gates are reversible.



Statement: The above quantum circuits are identical such that they have the same truth table.

TRUE / FALSE

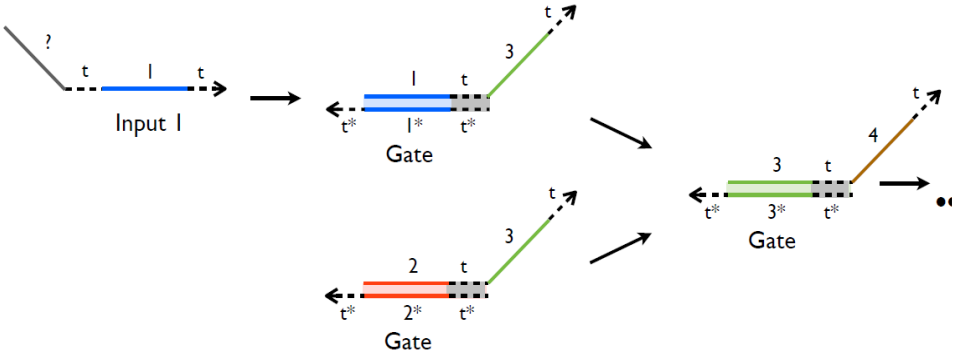
d) DNA strands tend to have larger attached (matched) parts. Consider the DNA in (a) shown below. Its matched part consists of 2 and 2*.



Statement: Adding the strand in (b) causes DNA strand displacement.

TRUE / FALSE

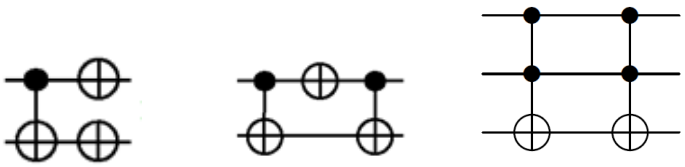
e) Consider an input DNA single strand and 3 DNA gates shown below.



Statement: After reactions take place, an output single stand "3-t-4-t" exists.

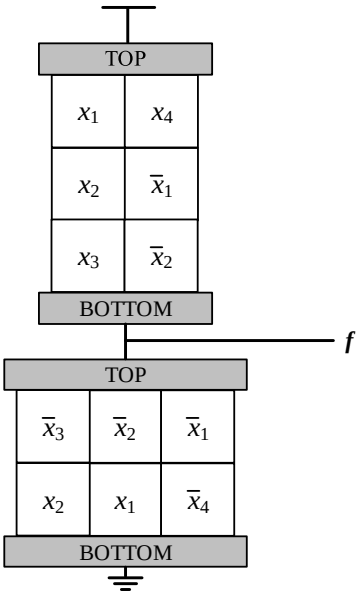
TRUE / FALSE

2) Consider the reversible circuits shown below. Simplify each of these circuits by using a fewer number of NOT, CNOT, and CCNOT (Toffoli) gates, if you can.

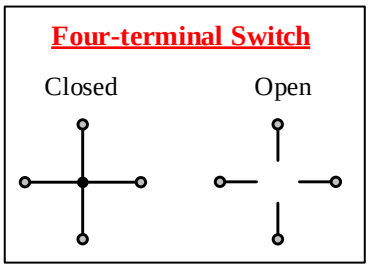


a) b) c)

3) Find top-to-bottom Boolean functions of the two lattices/arrays shown below. Find the Boolean expression of the output function ***f***.



A circuit consisting of four-terminal switches.



Four-terminal switch; closed if the controlling literal is 1*; open otherwise.*

4) Find expressions for the amount of $\boldsymbol{c}$ ($|\boldsymbol{c}|$) in terms of $|\boldsymbol{a}|$ and $|\boldsymbol{b}|$ for each case. Suppose that only the molecules $\boldsymbol{a}$ and $\boldsymbol{b}$ (sufficient amount more than 100) are present initially.

case 1

$$5a \xrightarrow{\text{fast}} 4a + b$$
$$a \xrightarrow{\text{slowest}} x$$
$$x + b \xrightarrow{\text{fastest}} x + y + c$$
$$x \xrightarrow{\text{fast}} 0$$
$$y \xrightarrow{\text{slow}} b$$

case 2

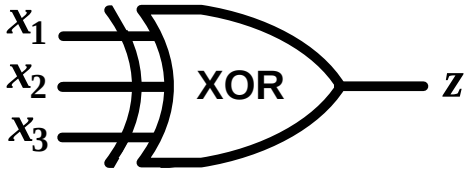
$$3a + 3b \xrightarrow{\text{fast}} 2a + 2b + x$$
$$a \xrightarrow{\text{slow}} 2c$$
$$b \xrightarrow{\text{slow}} 2c$$
$$x \xrightarrow{\text{slow}} 4c$$

5) Consider an XOR-3 gate shown below with stochastic inputs and output $\boldsymbol{x_1, x_2, x_3}$ and $\boldsymbol{z}$.

a) Derive a **stochastic** expression of $\boldsymbol{z}$ in terms of the inputs $\boldsymbol{x_1}$, $\boldsymbol{x_2}$ and $\boldsymbol{x_3}$. Note that z, x_1, x_2 and x_3 all represent probability values.

b) Suppose that **randomly shuffled** bit streams of 4 bits are applied to the inputs such that $x_1 = \frac{1}{4}$, $x_2 = \frac{2}{4}$, and $x_3 = \frac{3}{4}$. For example, 1,0,0,0 for x_1, 1,0,1,0 for x_2, and 1,0,1,1 for x_3. Using the expression found in **a)**, find the expected value of $\boldsymbol{z}$, and name this value $\boldsymbol{z_{exp}}$. Error at the output is defined: $\boldsymbol{Error} = \frac{|z - z_{exp}|}{z_{exp}}$. Find the **average** error at the output by considering all different possibilities for the input bit streams. Note that error at the output depends on how 1s and 0s are distributed in input bit streams.

c) If the gate has a permanent defect and it always gives the complement of the correct Boolean value (works like an XNOR gate) then what is the average error probability at the output?

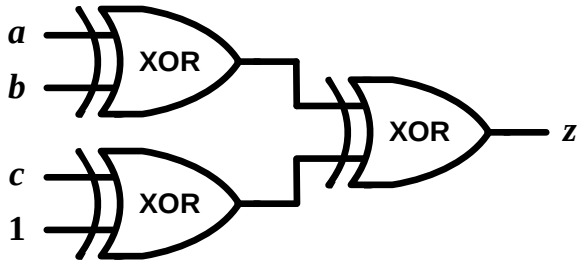


An XOR-3 gate

6) Consider the circuit shown below.

a) Suppose that each gate independently produces the complement of the correct Boolean value (**transition error**) with the same error probability of ϵ**.** Find the probability that the output **z** produces an incorrect result. Expressions should be in terms of ϵ and/or the inputs. In your expressions, use arithmetic operators (not Boolean).

b) Suppose that each gate independently produces a Boolean value of 0 at the output (**stuck-at 0**) with the same error probability of ϵ**.** Find the probability that the output **z** produces an incorrect result. Expressions should be in terms of ϵ and/or the inputs. In your expressions, use arithmetic operators (not Boolean).



Circuit with faulty gates.